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T. B. C. : PGT – 5/21

Test Booklet Series

A

TEST BOOKLET

PAPER – II

MATHEMATICS

50349

Sl. No.

Time Allowed : 2 Hours | *Maximum Marks : 100*

: INSTRUCTIONS TO CANDIDATES :

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET OF THE SAME SERIES ISSUED TO YOU.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C OR D**, AS THE CASE MAY BE, IN THE APPROPRIATE PLACE IN THE ANSWER SHEET USING BALL POINT PEN (BLUE OR BLACK).
3. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write *anything else* on the Test Booklet.
4. **YOU ARE REQUIRED TO FILL UP & DARKEN ROLL NO., TEST BOOKLET / QUESTION BOOKLET SERIES IN THE ANSWER SHEET AS WELL AS FILL UP TEST BOOKLET / QUESTION BOOKLET SERIES AND SERIAL NO. AND ANSWER SHEET SERIAL NO. IN THE ATTENDANCE SHEET CAREFULLY. WRONGLY FILLED UP ANSWER SHEETS ARE LIABLE FOR REJECTION AT THE RISK OF THE CANDIDATE.**
5. This Test Booklet contains **100** items (questions). Each item (question) comprises four responses (answers). You have to select the correct response (answer) which you want to mark (darken) on the Answer Sheet. In case, you feel that there is more than one correct response (answer), you should mark (darken) the response (answer) which you consider the best. In any case, choose **ONLY ONE** response (answer) for each item (question).
6. You have to mark (darken) all your responses (answers) **ONLY** on the **separate Answer Sheet** provided, by **using BALL POINT PEN (BLUE OR BLACK)**. See instructions in the Answer Sheet.
7. All items (questions) carry equal marks. All items (questions) are compulsory. Your total marks will depend only on the number of correct responses (answers) marked by you in the Answer Sheet. **There will be no negative markings for wrong answers.**
8. Before you proceed to mark (darken) in the Answer Sheet the responses (answers) to various items (questions) in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per the instructions sent to you with your **Admission Certificate**.
9. After you have completed filling in all your responses (answers) on the Answer Sheet and after conclusion of the examination, you should hand over to the Invigilator the *Answer Sheet* issued to you. You are allowed to take with you the candidate's copy / second page of the Answer Sheet along with the **Test Booklet**, after completion of the examination, for your reference.
10. Sheets for rough work are appended in the Test Booklet at the end.

SEAL

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1. The dimension of the vector space $V = \{M = (a_{ij})_{n \times n} : a_{ij} \in \mathbb{C}, a_{ij} = -a_{ji}\}$ over the field $\mathbb{R}$ is :

(A) $\frac{n^2-1}{2}$

(B) $\frac{n^2-n}{2}$

(C) $\frac{n^2+1}{2}$

(D) $\frac{n^2}{2}$

2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then A^{100} is :

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 100 & 1 & 0 \\ 100 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 98 & 1 & 0 \\ 98 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 49 & 1 & 0 \\ 49 & 0 & 1 \end{bmatrix}$

3. Consider the vector space $\mathbb{R}^3$ and the maps $f, g : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f(x, y, z) = (x, y, |z|)$ and $g(x, y, z) = (x, y+1, z-1)$. Then :

(A) g is linear but not f

(B) f is linear but not g

(C) Both f and g are linear

(D) Neither f nor g is linear

4. If the nullity of the matrix $\begin{bmatrix} m & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 2022 \end{bmatrix}$ is 1, then the value of m is :

(A) 0

(B) 1

(C) −1

(D) 2

5. If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{bmatrix}$, then the trace of A^{2022} is :

(A) 3

(B) 0

(C) 1

(D) 2

6. Let $T : \mathbb{R}^{20} \to \mathbb{R}^{20}$ be such that $T^7(x) = 0$, for all $x \in \mathbb{R}^{20}$, then :

(A) $6 \leq$ Nullity of $T \leq 12$

(B) $7 \leq$ Nullity of $T \leq 15$

(C) $8 \leq$ Nullity of $T \leq 14$

(D) None of these

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be such that $T(a, b)$ = the reflection of (a, b) in the line $y = \tan \alpha x$, then :

(A) $T(a, b) = (a \cos \alpha - b \sin \alpha, b \cos \alpha + a \sin \alpha)$

(B) $T(a, b) = (a \cos 2\alpha - b \sin 2\alpha, a \sin 2\alpha + b \cos 2\alpha)$

(C) $T(a, b) = (a \cos \alpha + b \sin \alpha, b \cos \alpha - a \sin \alpha)$

(D) $T(a, b) = (a \cos 2\alpha + b \sin 2\alpha, a \sin 2\alpha - b \cos 2\alpha)$

8. Let A be a 2 × 2 orthogonal matrix of trace and determinant 1. Then, the angle between Au and $u(u = [0, 1]^t)$ is :

(A) 60^o

(B) 90^o

(C) 30^o

(D) 45^o

9. Consider the following statements :

(i) $|\sin z| \leq 1$, for all $z \in \mathbb{C}$

(ii) $f(z) = |z|^2$ is analytic in $\mathbb{C}$

(A) Both (i) and (ii) are true

(B) Both (i) and (ii) are false

(C) Only (i) is true

(D) Only (ii) is true

10. Let C be the circle defined by $|z| = 3$ in the complex plane, described in the anti-clockwise direction. Then $\int_C \frac{2z^2 - z - 2}{z - 2} dz$ is :

(A) 0

(B) $4\pi i$

(C) 4π

(D) $8\pi i$

11. Let $E = \{x \in \mathbb{C} : |z| > 1\} \cup \{i\}$. Then :

(A) E is open but not closed in $\mathbb{C}$

(B) E is closed but not open in $\mathbb{C}$

(C) Neither open nor closed in $\mathbb{C}$

(D) Both open and closed in $\mathbb{C}$

12. Consider the following statements :

(i) Every analytic function in the extended plane is constant.

(ii) If f(z) is an entire function such that $f(z) = u + iv$ and $u^2 + v^2 \leq 2022$. Then f(z) is constant.

(A) Both (i) and (ii) are false

(B) Both (i) and (ii) are true

(C) Only (i) is true

(D) Only (ii) is true

13. The mapping $w = z^2 - 2z - 3$ is :

(A) Conformal everywhere

(B) Not conformal at $z = -1$ and $z = -3$

(C) Conformal with $|z| = 1$

(D) Not conformal at $z = 1$

14. The residue of the function $f(z) = \dfrac{1+2z}{z^2 + y^3}$ at y = 0 is :

(A) 0

(B) 1

(C) 2

(D) −1

15. The number of coefficient in a most general form of a homogeneous cubic which is harmonic as well is :

(A) 2

(B) 1

(C) 4

(D) 3

16. Principle value of i^i is :

(A) e

(B) 1/e

(C) $e^{-\frac{\pi}{2}}$

(D) $e^{\frac{\pi}{2}}$

17. The bilinear transformation which maps z = 1, i, 2 + i of Z plane onto the points w = i, 1, ∞ of W plane is :

(A) $\dfrac{(2+i)z-(2i+1)}{z-(2+i)}$

(B) $\dfrac{(2+i)z+(2i+1)}{z+(2+i)}$

(C) $\dfrac{(2-i)z+(2i-1)}{z+(2-i)}$

(D) $\dfrac{(2-i)z-(2i-1)}{z-(2-i)}$

18. If α is an eigen value of nonsingular matrix $A_{n\times n}$, then an eigenvalue of Adj(A) is :

(A) $\frac{1}{\alpha}$

(B) α

(C) α^n

(D) $\frac{|A|}{\alpha}$

19. Let $A = \begin{bmatrix} 1 & -2020 & -2019 \\ 0 & -2 & 2021 \\ 0 & 0 & -1 \end{bmatrix}$, then |Adj Adj Adj (A)| (Adj (A) = adjoint of A) is :

(A) 512

(B) 64

(C) 256

(D) 729

20. Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$, then the characteristic polynomial of A is :

(A) $x^5 - x^4 - x^3 - x^2 - x - 1$

(B) $x^5 + x^4 + x^3 + x^2 + x + 1$

(C) $x^5 + x^4 - x^3 + x^2 - x - 1$

(D) $x^5 + x^4 + x^3 + x^2 + x - 1$

21. Let $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 1 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ and λ is an eigenvalue of A, then :

(A) $-6 \le \lambda \le 9$

(B) $6 \le \lambda \le 9$

(C) $7 \le \lambda \le 9$

(D) $0 \le \lambda \le 9$

22. If $A = \begin{bmatrix} 1+s & -s \\ s & 1-s \end{bmatrix}$, then e^A is :

(A) e

(B) A

(C) Ae

(D) Ae + I

23. The bilinear transformation $w = \frac{2019z + 2020}{2021z + 2022}$ is conformal at :

(A) $z = 0$

(B) $z = -\infty$

(C) $z = 2$

(D) $z = \infty$

24. The image of the closed half disk $|z| \le 1$, $\mathrm{Im}(z) \ge 0$ under the bilinear transformation $w = \frac{z}{1+z}$ is :

(A) $u \ge 1/2$ and $v \le 0$

(B) $u \le 1/2$ and $v \ge 0$

(C) $u \le 0$ and $v \ge 1/2$

(D) $u \ge 0$ and $v \le 1/2$

25. The integral $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ is :

(A) 2π

(B) π

(C) $\pi/2$

(D) $\pi^2/4$

26. The integral $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta)\, d\theta$ is :

(A) 2π

(B) π

(C) $\pi/2$

(D) None of these

27. The Laplace transformation of $L[x \cos ax]$ is :

(A) $\frac{2as}{(s^2+a^2)^2}$

(B) $\frac{s^2-a^2}{(s^2+a^2)^2}$

(C) $\frac{as}{(s^2+a^2)^2}$

(D) $\frac{s^2+a^2}{(s^2+a^2)^2}$

28. Let $p(x-a) = \begin{cases} 0 & \text{for } x < a \\ 3 & \text{for } x \ge a \end{cases}$ Then $L[p(x-a)]$ is :

(A) $\frac{3e^{-sa}}{s}$

(B) $\frac{e^{-sa}}{s}$

(C) $\frac{3e^{sa}}{s}$

(D) $\frac{e^{-sa}}{3s}$

29. The Laplace transformation of Unit Impulse function is :

(A) $e^{-sa/3}$

(B) $e^{sa/3}$

(C) e^{-sa}

(D) $3e^{sa}$

30. If $y'' + 2y' + y = 1$ such that $y(0) = 2$ and $y'(0) = -2$. Then $L[y]$ is :

(A) $\frac{1}{s} - \frac{1}{s-1} + \frac{1}{(s-1)^2}$

(B) $\frac{1}{s} + \frac{1}{s-1} - \frac{1}{(s-1)^2}$

(C) $\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$

(D) $\frac{1}{s} + \frac{1}{s+1} - \frac{1}{(s+1)^2}$

31. Suppose a group contains elements a and b such that $|a| = 4$, $|b| = 2$ and $a^3b = ba$. Then $|ab|$ is :

(A) 2

(B) 4

(C) 8

(D) 1

32. The number of abelian groups (upto isomorphism) of order 360 is :

(A) 4

(B) 6

(C) 8

(D) 10

33. Let G be any group and H be a normal subgroup of G. In the factor group G/H given that $aH = bH$, then :

(A) $|a|$ divides $|b|$

(B) $|a| = |b|$ always

(C) $\exists a, b$ such that $|a| \neq |b|$

(D) $|b|$ divides $|a|$

34. Suppose that ϕ is a homomorphism from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{30}$ and Ker $\phi = \{0, 10, 20\}$. If $\phi(23) = 9$, then $\phi(9)$ is the set :

(A) {13, 23}

(B) {23}

(C) {3, 13}

(D) {3, 13, 23}

35. The number of onto homomorphisms from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{10}$ is :

(A) 10

(B) 8

(C) 6

(D) 4

36. Let $G = GL(2, \mathbb{R})$ and let K be a subgroup of $\mathbb{R}^*$. Then $H = \{A \in G \mid \det(A) \in K\}$ is :

(A) A cyclic subgroup of G

(B) A normal subgroup of G

(C) Only a subgroup of G

(D) Not a normal subgroup of G

37. Let R be a ring of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ and $I = \{f(x) \in \mathbb{R} \mid f(0) = 0\}$. Then :

(A) I is a maximal ideal of R

(B) I is not a maximal ideal of R

(C) I is both prime and maximal ideal of R

(D) I is maximal but not a prime ideal of R

38. Singular solution of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{dy}{dx}\right)x - y = 0$ is :

(A) $c^3 + cx - y = 0$

(B) $27y^3 + 4x^2 = 0$

(C) $4x^3 + 27y^2 = 0$

(D) $y = 5x + 125$

39. Let $a_2(x)y'' + a_1(x)y' + a_0y = 0$ be a second order differential equation, and k_1 and k_2 be the solution of its indicial equation such that $k_1 = k_2$. Then $y = Ay_1 + By_2$ is the complete solution if :

(A) $y_1 = (y)_{k = k_1(=k_2)}$ and $y_2 = \left(\frac{\partial y}{\partial x}\right)_{k=k_1(=k_2)}$

(B) $y = (y)_{k = k_1(=k_2)} = Ay_1 + By_2$

(C) $y_1 = (y)_{k = k_1(=k_2)}$ and $y_2 = \left(\frac{\partial y}{\partial k}\right)_{k=k_1(=k_2)}$

(D) None of these

40. Let $y_1(x) = 1 - x$ and $y_2(x) = e^x$ be two solutions of $y''(x) + P(x)y'(x) + Q(x)y(x) = 0$. Then $P(x) =$:

(A) $\frac{1-x}{x-2}$

(B) $-\left(\frac{1+x}{x}\right)$

(C) $\frac{x}{x-1}$

(D) $\frac{x-2}{x-1}$

41. The eigenvalue of the boundary value problem $y''(x) + \lambda y(x) = 0$, $y(0) = 0$ and $y(\pi) + y'(\pi) = 0$ is :

(A) $\lambda + \tan\sqrt{\lambda}\pi = 0$

(B) $\sqrt{\lambda} + \tan\lambda\pi = 0$

(C) $\lambda + \tan\lambda\pi = 0$

(D) $\sqrt{\lambda} + \tan\sqrt{\lambda}\pi = 0$

42. Let M be 2×2 matrix with real entries. Consider the linear system of ODE given in the vector rotation $\frac{dx(t)}{dt} = Mx(t)$ and $x(t) = [u(t), v(t)]^T$, then pick out the case when $\lim_{t\to\infty} u(t) = 0$, $\lim_{t\to\infty} v(t) = 0$:

(A) $M = \begin{bmatrix} -5 & 11 \\ 0 & -7 \end{bmatrix}$

(B) $M = \begin{bmatrix} 2 & 7 \\ 0 & 3 \end{bmatrix}$

(C) $M = \begin{bmatrix} 5 & 0 \\ -51 & 3 \end{bmatrix}$

(D) $M = \begin{bmatrix} -5 & 5 \\ 1 & -7 \end{bmatrix}$

43. Let u(x, t) be the solution of $u_{tt} - u_{xx} = 1, x \in \mathbb{R}, t > 0$ with $u(x, 0) = 0$, $u_t(x, 0) = 0, x \in \mathbb{R}$. Then, $u\left(\frac{1}{3}, \frac{1}{3}\right)$ is equal to :

(A) $\frac{1}{9}$

(B) $-\frac{1}{9}$

(C) $-\frac{1}{18}$

(D) $\frac{1}{18}$

44. The initial value problem $u_x + u_y = 1$, $u(s, s) = \sin s, 0 \le s \le 1$ has :

(A) No solution

(B) Infinitely many solutions

(C) A unique solution

(D) Two solutions

45. Let the partial differential equation $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$ satisfying the initial condition $u(x, 0) = \delta + \lambda x$. If $u(x, t) = 1$ along the characteristic $x = t + 1$, then :

(A) $\delta = 2, \lambda = 0$

(B) $\delta = 1, \lambda = 1$

(C) $\delta = 0, \lambda = 1$

(D) $\delta = 0, \lambda = 0$

46. The solution of Cauchy problem

$u_{yy}(x, y) - u_{xx}(x, y) = 0$; $u(x, 0) = 0$, $u_y(x, 0) = x$ is $u(x, y) = ?$

(A) $xy + \frac{x}{y}$

(B) xy

(C) $\frac{x}{y}$

(D) None of these

47. The unique solution of the system of three congruences

$x \equiv 2 \pmod 3$

$x \equiv 3 \pmod 5$

$x \equiv 2 \pmod 7$

is :

(A) $233 \equiv 23 \pmod{105}$

(B) $233 \equiv 23 \pmod 3$

(C) $233 \equiv 23 \pmod 5$

(D) $233 \equiv 23 \pmod 7$

48. The system of linear congruences

$7x + 3y \equiv 10 \pmod{16}$

$2x + 5y \equiv 9 \pmod{16}$

has :

(A) Unique solution

(B) Two solutions

(C) Infinitely many solutions

(D) No solution

49. Let p be a prime, consider the following two statements :

(i) $p/n^p + (p-1)!\,n$ for any integer n.

(ii) $p/(p-1)!\,n^p + n$ for any integer n.

Then :

(A) Only (i) is true

(B) Only (ii) is true

(C) Both (i) and (ii) are true

(D) None of these

50. Let V = (G, E) be a directed graph. Then which of the following has the same connected component as G has :

(A) $G_1 = (V, E_1)$, where $E_1 = \{uv \in E_1 \mid vu \notin E\}$

(B) $G_2 = (V, E_2)$, where $E_2 = \{uv \in E_2 \mid vu \in E\}$

(C) $G_3 = (V, E_3)$, where $E_3 = \{uv \in E_3\}$ if there is a path of length ≤ 2 between u and v

(D) $G_4 = (V_4, E)$, V_4 is a set of vertices which are not isolated

51. In a connected graph, which of the following is true ?

(A) Every regular graph is not connected.

(B) Every pair of vertices may not have a path between them.

(C) A bridge cannot be a part of simple cycle.

(D) A graph with bridges cannot have a cycle.

52. The maximum number of edges in a bipartite graph of order 12 is :

(A) 18

(B) 36

(C) 12

(D) None of these

53. A graph is cyclic with n vertices and is isomorphic to its complement. Then the value of n is :

(A) 4

(B) 3

(C) 2

(D) 5

54. The number of edges in a regular graph of degree d and n vertices is :

(A) max {n , d}

(B) n + d

(C) nd

(D) nd/2

55. The maximum degree of any vertex in a simple graph with n vertices is :

(A) n

(B) n + 1

(C) n – 1

(D) 2n

56. How many non-isomorphic graphs are possible with 6 vertices and 6 edges and degree of each vertex is 2 ?

(A) 2

(B) 4

(C) 6

(D) 8

57. A non-trivial graph G is bipartite if :

(A) G does not contain an odd cycle

(B) G contains an odd cycle

(C) G does not contain an even cycle

(D) G contains an even cycle

58. If uv is a bridge in a graph G, then :

(A) There does not exist u – v path in G

(B) There are many u – v paths in G

(C) There is a unique u – v path in G

(D) None of these

59. Which of the followings are Eulerian graphs ?

(A) Cycle C_n for $n \geq 3$

(B) Complete graph with vertices $n \geq 3$ and n is odd

(C) $K_{m,n}$ where both m, n are even

(D) All of these

60. In full binary search tree every internal node has exactly two children. If there are 100 leaf nodes in the tree, how many internal nodes are there in the tree ?

(A) 25

(B) 49

(C) 99

(D) 101

61. Which of the following statement is false ?

(A) Every tree is a bipartite graph.

(B) A tree contains a cycle.

(C) A tree with n nodes contains n – 1 edges.

(D) A tree is a connected graph.

62. Which of the following is false ?

(A) Any product of compact space is compact.

(B) Any product of metrizable space is metrizable.

(C) Any product of connected space is connected.

(D) Any product of Hausdorff space is Hausdorff.

63. Under usual topology in $\mathbb{R}^3$, if P = $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 < 1\}$ and Q = $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 = 0\}$, then $P \cap Q$ is :

(A) Closed but not open

(B) Open but not closed

(C) Both open and closed

(D) Neither open nor closed

64. Let X and Y be topological spaces and $f : X \to Y$ be a continuous and bijective map. Then, f is a homeomorphism, if :

(A) X and Y are Hausdorff

(B) X is Hausdorff and Y is compact

(C) X is compact and Y is Hausdorff

(D) X and Y are compact

65. Let X be the indiscrete space and Y is a T_0 space. If $f : X \to Y$ is continuous, then :

(A) X must be one-point space

(B) Y must be one-point space

(C) Y must be discrete

(D) f must be a constant

66. A metric space is always :

(A) Separable

(B) Lindelof

(C) First countable

(D) Second countable

67. In $Z_5[x]$, let $I = \langle x^2 + x + 2 \rangle$, then the multiplicative inverse of $2x + 3 + I$ in $Z_5[x]/I$ is :

(A) $3x + 1 + I$

(B) $3x + 2 + I$

(C) $4x + 1 + I$

(D) $4x + 3 + I$

68. Let $f(x, y) = x^3 + y^3 - 63(x + y) + 12(xy)$. Then :

(A) At (5, –1) f has a maximum

(B) At (5, –1) f has a minimum

(C) At (5, –1) f has neither maximum nor minimum

(D) At (–1, 5) f has a maximum

69. Let $u = f(x, y)$ and $v = g(x, y)$ have continuous partial derivatives in a region R of the xy-plane. A necessary and sufficient condition that they satisfy a functional relation, say $F(u, v) = 0$ is that the Jacobian :

(A) $\frac{\partial(u,v)}{\partial(x,y)} \neq 0$

(B) $\frac{\partial(u,v)}{\partial(x,y)} = 0$

(C) $\frac{\partial(u,v)}{\partial(x,y)}$ is not defined

(D) None of these

70. If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$, then the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ is equal to :

(A) $r^2 \sin\theta$

(B) $r^2 \cos\theta$

(C) $r \sin\theta$

(D) $r \cos\theta$

71. Which of the following function is not uniformly continuous on (0, 1) ?

(A) x^2

(B) $\frac{1}{x^2}$

(C) $\sin x$

(D) $\frac{\sin x}{x}$

72. Let s_n be a sequence of real numbers on a bounded set S, where $\lim\inf s_n \neq \lim\sup s_n$. Which of the following is not necessarily true ?

(A) $\lim s_n$ does not exist

(B) $\lim\inf s_n < \lim\sup s_n$

(C) There exists a convergent subsequence

(D) s_n has infinite number of dominant terms

73. Which of the following is not true about $s_n = \frac{1}{n}$?

(A) $\lim\sup s_n = 0$

(B) The series $\sum(-1)^n s_n$ converges

(C) The series $\sum s_n^2$ converges

(D) $\lim_{n\to\infty} \sum_{i=1}^{n} s_i = L$, for some finite L

74. Which of the following series is convergent ?

(A) $\sum \frac{x^n}{n!}$ for all x

(B) $\sum \frac{1}{n+\sin(n)}$

(C) $\sum (-1)^n n$

(D) $\sum \sin n$

75. Let $f(x, y) = 4x^2 - xy + 4y^2 + x^3y + xy^3 - 4$. Then :

(A) (0, 0) is the only stationary point

(B) At (0, 0) f has a minimum and $f_{min} = -4$

(C) At (0, 0) f has a minimum and $f_{min} = 0$

(D) The point $\left(\frac{3}{2}, -\frac{3}{2}\right)$ is a point of extrema

76. Let f be a differentiable function, where all derivatives exist, such that $f(0) = 0$, $f'(0) = 0$ and $|f''(x)| \le M$ for all x. Which of the following is not necessarily true ?

(A) $f(1) \le \frac{M}{2}$

(B) 0 is neither a maximum nor a minimum

(C) $\forall \in > 0, \exists\, \delta > 0$ such that if $x \in (-\delta, \delta)$, then $|f(x)| < \in$

(D) None of these

77. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Then :

(A) f is differentiable everywhere

(B) f is continuous at (0, 0) but not differentiable at (0, 0)

(C) f is not continuous at (0, 0)

(D) f is differentiable only at (0, 0)

78. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be such that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at all points. Then :

(A) All directional derivatives of f exists at all points of $\mathbb{R}^2$

(B) The total derivation of f exists at all points of $\mathbb{R}^2$

(C) The function f(x, y) as a function of x for every fixed y and f(x, y) as a function of y for every fixed x are continuous

(D) f is continuous on $\mathbb{R}^2$

79. Let $f(x, y) = \sqrt{|xy|}$, then the value of $f_x(0, 0)$ and $f_y = (0, 0)$ is :

(A) 0, 0

(B) 1, 0

(C) 1, 1

(D) 0, 1

80. Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$. Using Stoke's theorem the state of the function will be :

(A) Solenoidal

(B) Divergent

(C) Rotational

(D) Irrotational

81. The surface integral $\iint \frac{1}{\pi}(9x\hat{i} - 3y\hat{j}) \cdot \hat{n} ds$ over the sphere $x^2 + y^2 + x^2 = 9$ is :

(A) 213

(B) 214

(C) 215

(D) 216

82. The volume under the surface $z(x, y) = x + y$ and the triangle bounded by the line $x = y$, $x = 0$, $y = 1$ in the xy-plane defined by $\{0 \le y \le x$ and $0 \le x \le 12\}$ is :

(A) 846

(B) 864

(C) 684

(D) 648

83. Let $\{E_i\}$ be a sequence of measurable sets :

(i) If $E_1 \subseteq E_2 \subseteq \ldots$, we have $m(\lim E_i) = \lim m(E_i)$.

(ii) If $E_1 \supseteq E_2 \supseteq \ldots$, and $m(E_i) < \infty$ for each i, then we have $m(\lim E_i) = \lim m(E_i)$.

Then :

(A) Only (i) is true

(B) Only (ii) is true

(C) Both (i) and (ii) are true

(D) Neither (i) nor (ii) is true

84. Let $\{E_i\}$ be any sequence of sets. Then :

(A) $m*\left(\bigcup_{i=1}^{\infty} E_i\right) \le \sum_{i=1}^{\infty} m*(E_i)$

(B) $m*\left(\bigcup_{i=1}^{\infty} E_i\right) \ge \sum_{i=1}^{\infty} m*(E_i)$

(C) $m*\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m*(E_i)$

(D) None of these

85. Let f be any extended real-valued function such that $f(x + y) = f(x) + f(y)$ for every x and y. If f is measurable and finite, then for each x, we have :

(A) $f(x) = 0$

(B) $f(x) = 1$

(C) $f(x) = x f(0)$

(D) $f(x) = x f(1)$

86. Let f be a function defined on (0, 1) by $f(x) = \begin{cases} 0, & x \text{ is rational} \\ [1/x]^{-1}, & x \text{ is irrational} \end{cases}$ where [x] = integer part of x. Then $\int_0^1 f\,dx =$:

(A) 0

(B) 1

(C) 2

(D) ∞

87. For what value of α and β, the quadrature formula $\int_{-1}^{1} f(x)dx \approx \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree ≤ 1?

(A) $\alpha = 1, \beta = 1$

(B) $\alpha = 1, \beta = -1$

(C) $\alpha = -1, \beta = 1$

(D) $\alpha = -1, \beta = -1$

88. One root of the equation $e^x - 3x^2 = 0$ lies in the interval (3, 4). The least number of iterations of the bisection method, so that $|error| \leq 10^{-3}$ are :

(A) 8

(B) 10

(C) 6

(D) 2

89. To find the positive square root of $a > 0$ by solving $x^2 - a = 0$ by the Newton-Raphson method, if x_n denotes the nth iterate with $x_0 > 0, x_0 \neq \sqrt{a}$, then the sequence $\{x_n, n \geq 1\}$ is :

(A) Constant

(B) Strictly decreasing

(C) Strictly increasing

(D) Not convergent

90. The linear programming problem

Max $z = x_1 + x_2$

s.t. $x_1 + 2x_2 \leq 20$

$x_1 + x_2 \leq 15$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

(A) Has exactly one optimum solution

(B) Has unbounded solution

(C) Has no solution

(D) Has more than one optimum solutions

91. The dual of the Linear Programming Problem

$\min c^T x$

s. t. $Ax \geq b$

$x \geq 0$

is :

(A) $\max b^T w$ s.t. $A^T w \geq c$ and $w \geq 0$

(B) $\max b^T w$ s.t. $A^T w \geq c$ and w is unrestricted

(C) $\max b^T w$ s.t. $A^T w \leq c$ and w is unrestricted

(D) $\max b^T w$ s.t. $A^T w \leq c$ and $w \geq 0$

92. If the cost matrix for an assignment problem is given by

$$\begin{bmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{bmatrix}$$

where a, b, c, d > 0, then the value of the assignment problem is :

(A) 4 min {a, b, c, d}

(B) min {a, b, c, d}

(C) max {a, b, c, d}

(D) a + b + c + d

93. The value of the integral $\int_{(a,b)\cap Q} k\,dx$ (where a, b is any real number and k is any constant) is :

(A) k (b – a)

(B) b – a

(C) k

(D) 0

94. Let $\mathbb{C}$ be the set of all complex numbers. Define :

(i) $d_1(z_1, z_2) = \min\{1, |z_1 - z_2|\}$ for all $z_1, z_2 \in \mathbb{C}$

(ii) $d_2(z_1, z_2) =$

$$\begin{cases} 0 & \text{if } z_1 = z_2 \\ |z_1| + |z_2| & \text{if } z_1 \neq z_2 \end{cases}$$

(iii) $d_3(z_1, z_2) =$

$$\begin{cases} \min\{|z_1| + |z_2|, |z_1 - 1| + |z_2 - 1|\} & \text{if } z_1 \neq z_2 \\ 0 & \text{otherwise} \end{cases}$$

Then :

(A) (i) and (iii) define metric on $\mathbb{C}$

(B) (ii) and (iii) define metric on $\mathbb{C}$

(C) All (i), (ii) and (iii) define metric on $\mathbb{C}$

(D) Only (ii) define metric on $\mathbb{C}$

95. Let (X, d_m) be a metric space, where X is an infinite set and for each positive integer m, the metric is defined as $d_m(x, y) = \begin{cases} 0 & x = y \\ m & x \neq y \end{cases}$

Then X is :

(A) Compact

(B) Not compact

(C) Not complete

(D) Not first countable

96. Let X and Y be two metric spaces, and $f : X \to Y$ be a continuous function. Then f^{-1} is continuous on Y if :

(A) f is bijective

(B) Y is compact

(C) f is bijective and Y is compact

(D) f is bijective and X is compact

97. Which of the following is not correct ?

(i) The set $\mathbb{N}$ of all natural numbers with usual metric induced by the usual metric of $\mathbb{R}$ is not connected.

(ii) The subset $A = \{x \in \mathbb{R} : |x| > 0\}$ of $\mathbb{R}$ is connected.

(iii) The subset $A = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 \geq 4\}$ of $\mathbb{R}^2$ is disconnected.

(iv) Every open or closed sphere in $\mathbb{R}^n$ is connected.

(A) (i)

(B) (ii)

(C) (iii)

(D) (iv)

98. Consider the Banach $C[0, \pi]$ with the sup norm. The norm of the linear functional $p : C[0, \pi] \to \mathbb{R}$ given by $p(f) = \int_0^{\pi} f(x) \sin^2 x \, dx$ is :

(A) $\frac{\pi}{2}$

(B) π

(C) 2π

(D) 1

99. Let $X = C[-1, 1]$ with the inner product defined by $\langle x, y \rangle = \int_{-1}^{1} x(t)\, y(t)\, dt$. Let Y be the set of all odd functions in X. Then :

(A) $Y^{\perp} = \{0\}$

(B) $Y^{\perp}$ is the set of all even functions in X

(C) $Y^{\perp}$ is the set of all odd functions in X

(D) $Y^{\perp}$ is the set of all constant functions in X

100. Let the continous linear operator $T : l^2 \rightarrow l^2$ be defined by $T(x_1, x_2, \ldots) = (0, x_1, 0, x_3, 0, x_5, 0, \ldots)$.

Then :

(A) T is compact but not T^2

(B) T^2 is compact but not T

(C) Both T and T^2 are compact

(D) Neither T nor T^2 is compact

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SPACE FOR ROUGH WORK

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